Ordinary partial derivatives (∂_μ) commute, but covariant derivatives (∇) do not necessarily commute.

$$\nabla\nabla f = \nabla(e^\mu\,\partial_\mu f)$$
$$= (\nabla e^\mu)(\,\partial_\mu f) + \cancel{e^\nu e^\mu \partial_\nu \partial_\mu f}$$
$$= S^\mu\,\partial_\mu f$$

We have defined the torsion $S^\mu = \nabla e^\mu$. The torsion has a vector part s and a spin-2 part S.

$$s = e_\mu S^\mu = e_\mu \nabla e^\mu \qquad S = e_\mu S^\mu = e_\mu \nabla e^\mu$$

Compute the torsion spinors from the connection spinors.

$$s = e_\mu \nabla e^\mu = e_\mu \partial e^\mu + e_\mu e^\mu \Gamma + e_\mu e^\mu \gamma$$
$$= e_\mu \partial e^\mu + \cancel{\Gamma} + \gamma$$
$$= e_\mu \partial e^\mu - \tfrac{3}{2}\gamma$$

$$S = e_\mu \nabla e^\mu = e_\mu \partial e^\mu + e_\mu e^\mu \Gamma + e_\mu e^\mu \gamma$$
$$= e_\mu \partial e^\mu + \Gamma + \cancel{\gamma}$$
$$= e_\mu \partial e^\mu + \Gamma$$

In General Relativity, torsion is zero. While torsion is an interesting extension, we will assume zero torsion from here on. This allows us to solve for the connection spinors in terms of the tetrad.

$$\gamma = \tfrac{2}{3} e_\mu \partial e^\mu \qquad \Gamma = -e_\mu \partial e^\mu$$

The vector connection γ appears to be real, which would mean the connection has fewer degrees of freedom than I thought.